

# Geometry: 7.1-7.3 Notes

NAME \_\_\_\_\_

## 7.1 Angles of Polygons

Date: \_\_\_\_\_

### Define Vocabulary:

diagonal

equilateral polygon

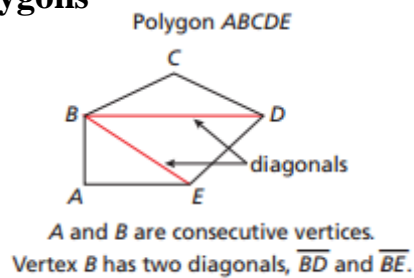
equiangular polygon

regular polygon

### Using Interior Angles Measures of Polygons

In a polygon, two vertices that are endpoints of the same side are called *consecutive vertices*.

A **diagonal** of a polygon is a segment that joins two nonconsecutive vertices.

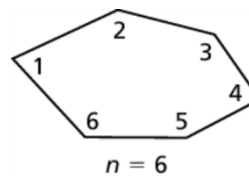


### Theorems

#### Theorem 7.1 Polygon Interior Angles Theorem

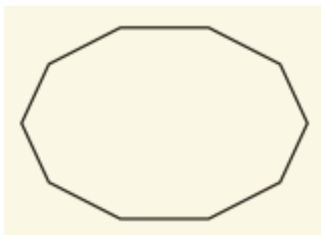
The sum of the measures of the interior angles of a convex  $n$ -gon is  $(n - 2) \cdot 180^\circ$ .

$$m\angle 1 + m\angle 2 + \cdots + m\angle n = (n - 2) \cdot 180^\circ$$



**Examples: Find the sum of the measures of the interior angles.**

1. **WE DO**



2. **YOU DO**

The coin shown is in the shape of an 11-gon



**Examples: Find the number of sides of the polygon.**

3. **WE DO**

The sum of the measures of the interior angles of a convex polygon is  $1800^\circ$ . Classify the polygon by the number of sides

4. **YOU DO**

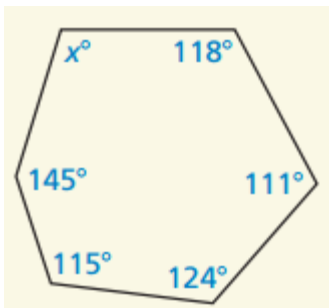
The sum of the measures of the interior angles of a convex polygon is  $2520^\circ$ . Classify the polygon by the number of sides

### Corollary 7.1 Corollary to the Polygon Interior Angles Theorem

The sum of the measures of the interior angles of a quadrilateral is  $360^\circ$ .

**Examples: Find the unknown interior angle measure.**

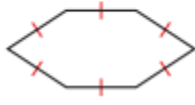
5. **WE DO**



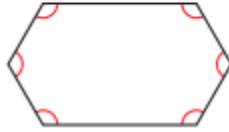
6. **YOU DO**

The measures of the interior angles of a quadrilateral are  $x^\circ$ ,  $3x^\circ$ ,  $5x^\circ$ , and  $7x^\circ$ . Find the measure of all the interior angles.

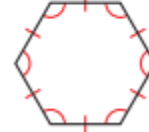
In an **equilateral polygon**, all sides are congruent.



In an **equiangular polygon**, all angles in the interior of the polygon are congruent.

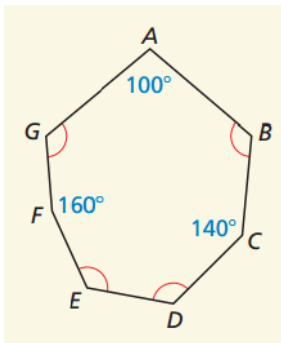


A **regular polygon** is a convex polygon that is both equilateral and equiangular.



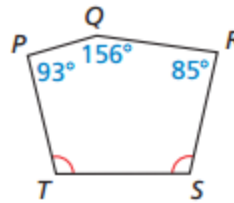
**Examples: Finding angle measures in polygons.**

7. **WE DO**



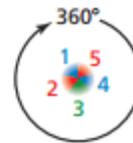
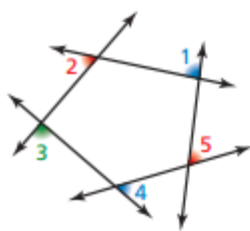
8. **YOU DO**

Find the measures of  $\angle S$  and  $\angle T$ .



a. Is the polygon regular? Explain your reasoning.

b. Find the measures of  $\angle B$ ,  $\angle D$ ,  $\angle E$ , and  $\angle G$ .

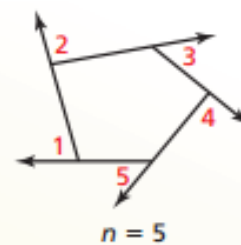


**Theorem 7.2 Polygon Exterior Angles Theorem**

The sum of the measures of the exterior angles of a convex polygon, one angle at each vertex, is  $360^\circ$ .

$$m\angle 1 + m\angle 2 + \dots + m\angle n = 360^\circ$$

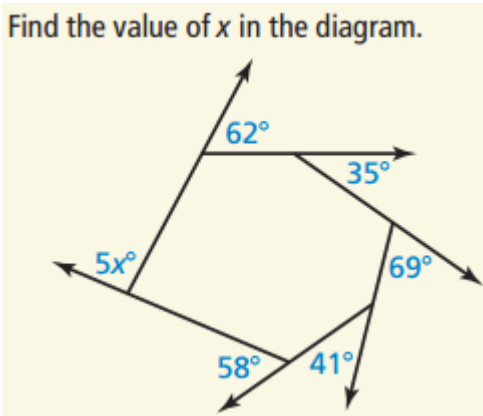
*Proof* Ex. 51, p. 366



**Examples: Finding an unknown exterior angle measure.**

9. **WE DO**

Find the value of  $x$  in the diagram.



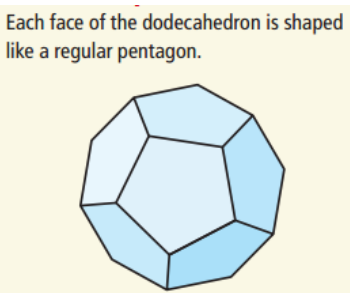
10. **YOU DO**

A convex hexagon has exterior angles with measures  $34^\circ$ ,  $49^\circ$ ,  $58^\circ$ ,  $67^\circ$ , and  $75^\circ$ . What is the measure of an exterior angle at the 6<sup>th</sup> vertex?

**Examples: Finding angle measure of regular polygons.**

11. **WE DO**

Each face of the dodecahedron is shaped like a regular pentagon.



a. Find the measure of each interior angle of a regular pentagon.

b. Find the measure of each exterior angle of a regular pentagon.

12. **YOU DO**

a. Find the measure of each interior angle and each exterior angle of a regular 24-gon.

b. Each exterior angle of a regular polygon has a measure of  $18^\circ$ . Find the number of sides of the regular polygon.

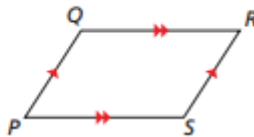
Assignment	
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## Define Vocabulary:

parallelogram

## Using Properties of Parallelograms

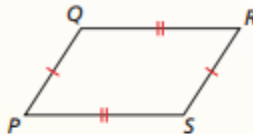
A **parallelogram** is a quadrilateral with both pairs of opposite sides parallel. In  $\square PQRS$ ,  $\overline{PQ} \parallel \overline{RS}$  and  $\overline{QR} \parallel \overline{PS}$  by definition. The theorems below describe other properties of parallelograms.

**Theorem 7.3 Parallelogram Opposite Sides Theorem**

If a quadrilateral is a parallelogram, then its opposite sides are congruent.

If  $\square PQRS$  is a parallelogram, then  $\overline{PQ} \cong \overline{RS}$  and  $\overline{QR} \cong \overline{PS}$ .

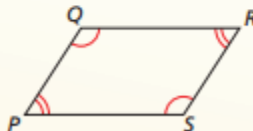
*Proof* p. 368

**Theorem 7.4 Parallelogram Opposite Angles Theorem**

If a quadrilateral is a parallelogram, then its opposite angles are congruent.

If  $\square PQRS$  is a parallelogram, then  $\angle P \cong \angle R$  and  $\angle Q \cong \angle S$ .

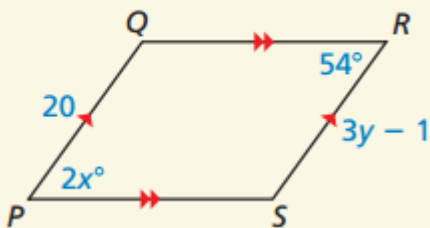
*Proof* Ex. 37, p. 373



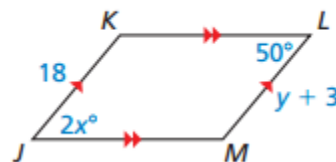
## Examples: Using properties of parallelograms.

1. **WE DO**

Find the values of  $x$  and  $y$ .

2. **YOU DO**

Find the values of  $x$  and  $y$ .

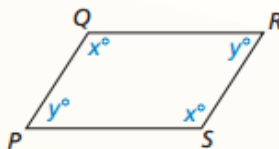


**Theorem 7.5 Parallelogram Consecutive Angles Theorem**

If a quadrilateral is a parallelogram, then its consecutive angles are supplementary.

If  $PQRS$  is a parallelogram, then  $x^\circ + y^\circ = 180^\circ$ .

*Proof* Ex. 38, p. 373

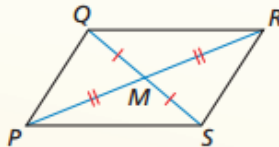


**Theorem 7.6 Parallelogram Diagonals Theorem**

If a quadrilateral is a parallelogram, then its diagonals bisect each other.

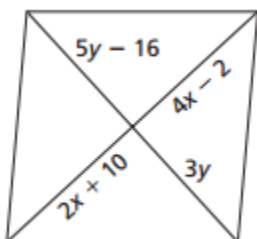
If  $PQRS$  is a parallelogram, then  $\overline{QM} \cong \overline{SM}$  and  $\overline{PM} \cong \overline{RM}$ .

*Proof* p. 370

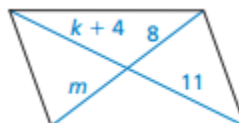


**Examples: Find the value(s) of the variable(s) in the parallelogram.**

3. **WE DO**



4. **YOU DO**



**Examples: Use the diagram to find the indicated measure.  $\square$  PMNO is a parallelogram.**

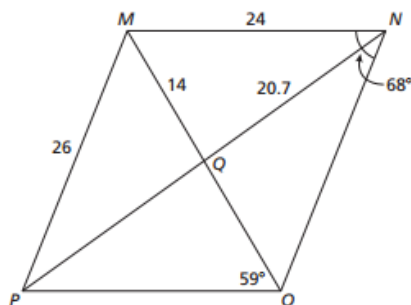
5. **WE DO**

a. PO

b. OQ

c.  $m\angle PMN$

d.  $m\angle NOP$



6. **YOU DO**

a. NO

b. PQ

c.  $m\angle OPM$

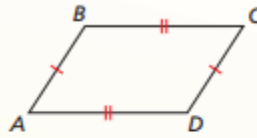
d.  $m\angle NMO$

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**Theorem 7.7 Parallelogram Opposite Sides Converse**

If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

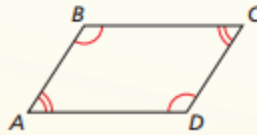
If  $\overline{AB} \cong \overline{CD}$  and  $\overline{BC} \cong \overline{DA}$ , then  $ABCD$  is a parallelogram.

**Theorem 7.8 Parallelogram Opposite Angles Converse**

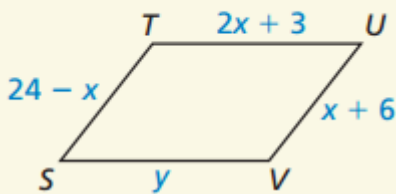
If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

If  $\angle A \cong \angle C$  and  $\angle B \cong \angle D$ , then  $ABCD$  is a parallelogram.

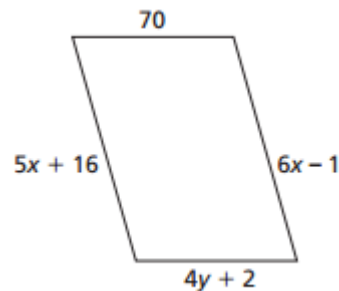
*Proof* Ex. 39, p. 383

**Examples: Finding side lengths of a parallelogram.**1. **WE DO**

For what values of  $x$  and  $y$  is quadrilateral  $STUV$  a parallelogram?

2. **YOU DO**

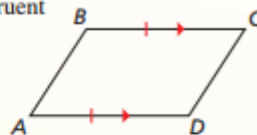
Find the values of  $x$  and  $y$  that make the quadrilateral a parallelogram.

**Theorem 7.9 Opposite Sides Parallel and Congruent Theorem**

If one pair of opposite sides of a quadrilateral are congruent and parallel, then the quadrilateral is a parallelogram.

If  $\overline{BC} \parallel \overline{AD}$  and  $\overline{BC} \cong \overline{AD}$ , then  $ABCD$  is a parallelogram.

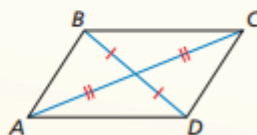
*Proof* Ex. 40, p. 383

**Theorem 7.10 Parallelogram Diagonals Converse**

If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

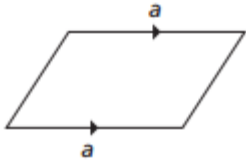
If  $\overline{BD}$  and  $\overline{AC}$  bisect each other, then  $ABCD$  is a parallelogram.

*Proof* Ex. 41, p. 383

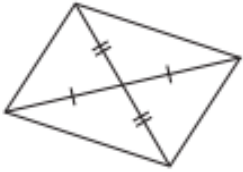


Examples: State which theorem you can use to show that the quadrilateral is a parallelogram.

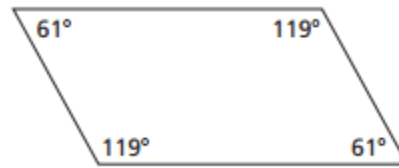
3. **WE DO**



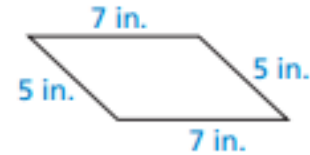
4.



5. **YOU DO**

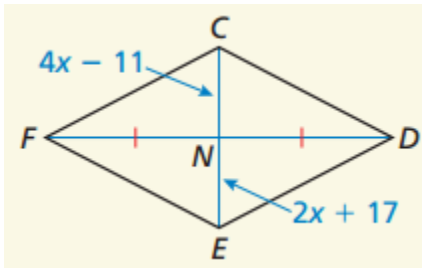


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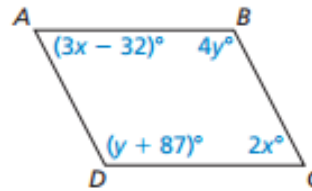


Examples: Find the value(s) of the variable(s) that make the quadrilateral a parallelogram.

7. **WE DO**



8. **YOU DO**



**Ways to Prove a Quadrilateral Is a Parallelogram**

1. Show that both pairs of opposite sides are parallel. ( <i>Definition</i> )	
2. Show that both pairs of opposite sides are congruent. ( <i>Parallelogram Opposite Sides Converse</i> )	
3. Show that both pairs of opposite angles are congruent. ( <i>Parallelogram Opposite Angles Converse</i> )	
4. Show that one pair of opposite sides are congruent and parallel. ( <i>Opposite Sides Parallel and Congruent Theorem</i> )	
5. Show that the diagonals bisect each other. ( <i>Parallelogram Diagonals Converse</i> )	

Assignment	
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